

## METHOD FOR ASSESSMENT OF MECHANICAL BEHAVIOUR OF STRUCTURES MADE OF RECYCLED CONCRETE

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### ABSTRACT

Application of recycled materials to everyday construction is hindered by the issues related to their often reduced mechanical properties, which the designers are very much aware of. Therefore, in the Czech Republic, most of the recycled materials are considered as good enough for use in backfills, road bases, occasionally roads and runways, and seldom for horizontal reinforced concrete structures, such as mere industrial floors. Vertical structural members are, as of now, out of question, which is also caused by the legislation, which does not favour use of recycled materials convincingly enough. However, the main reason is the limited knowledge about properties of recycled materials and their effect on ultimate performance of a structure, long-term behaviour, and most importantly on the economic feasibility, which is difficult to assess due to the scarce data. The proposed paper describes a method for structural analysis of reinforced concrete structures made of concrete with recycled aggregate. The limited data on the material parameters used in the modelling are expressed by fuzzy sets, which yield results also in the form of fuzzy sets, which then serve as input data for fuzzy-logic-based assessment of feasibility of the structure. This method is suitable for the preliminary design stage when it is decided whether or not a given project is worth pursuing. It is believed that such tools can help convince the designers about possible applicability of recycled building materials for structures of more than one floor, and quantify possible consequences. The method is illustrated in examples.

### KEYWORDS:

Recycled concrete; structural analysis; fuzzy set theory; assessment.

### INTRODUCTION

Partial substitution of the primary natural materials with recycled materials is the right trend. This approach is not only environment-friendly, but also in accordance with effective use of natural resources. In the Czech Republic, this issue is well covered in the document "Natural resources policy" approved by the government of the Czech Republic in December 13<sup>th</sup>, 1999. In the case of recycled building waste, this issue is more related to saving the natural resources and minimization of waste stored in dumping grounds. The most prospective use of recycled building materials is expected in the areas with shortage of natural resources, as the raw building materials, such as building stone, have little added value and thus their price is directly affected by the distance of transport. On the other hand, the designers are concerned with the reduced mechanical performance of the recycled building materials, which hinders their more common use.

This paper focuses on assessment of performance of concrete structural members and structures made of recycled aggregate. A method of assessment is briefly described and its applicability is documented in illustrative examples. Since the decision-making algorithm is based on the fuzzy set theory, some basic definition of a fuzzy set is given along with its extension to fuzzy numbers, which are used for description of uncertainty of material parameter in modelling of mechanical behaviour of recycled concrete structures. The fuzzy arithmetic is used in the structural analysis, which for more complex problems is run using the finite element method. The fuzzy logic is briefly introduced as it constitutes the backbone of the assessment method.

The ultimate objective of this paper is to help promote application of concrete with recycled aggregate. The quantitative information on performance of a structure, which is made of recycled concrete, and especially its price, may help convince the designers to use recycled materials also for multi-storey structures. This will in turn put pressure on scientific community to test and enhance such materials, which will trigger eventually the upward spiral of development.

## FUZZY SETS

The uncertainty, which is present in input parameters, can be handled with help of the fuzzy set theory (Zadeh, 1965), where the uncertain quantities are defined in terms of fuzzy sets. Unlike in the classical set theory, the membership of an element to a fuzzy set includes the values between 0 and 1, where 0 means "does not belong" and 1 means "definitely belongs" to a fuzzy set. Usually, the fuzzy sets represent vague verbal evaluation. In cases when a fuzzy set represents a numeral, it is called a fuzzy number.

### Fuzzy numbers

The notion of a fuzzy number arises from the experience of the everyday life when many phenomena which can be quantified are not characterized in the terms of absolutely precise numbers.

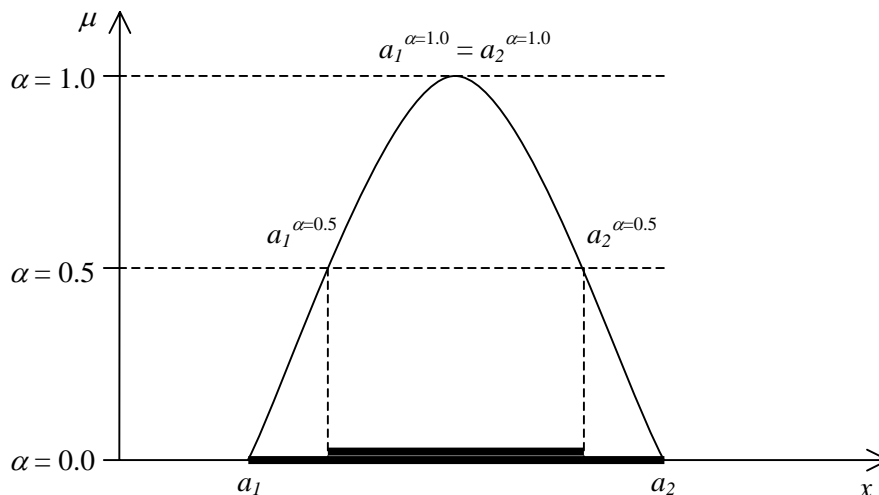


Figure 1: Normal fuzzy number and its  $\alpha$ -cuts

Fuzzy numbers are fuzzy sets which are defined on the set of real numbers. Their membership function assigns the degree of 1 to the central, also called nominal, modal or mean, value and lower degrees to other numbers which reflect their proximity to the central value according to the used membership function. The membership function should thus decrease from 1 to 0 on both sides of the central value. Such fuzzy sets are called fuzzy numbers. An example of a fuzzy number is shown in Figure 1, where  $\mu$  represents the membership function and  $a_1$  and  $a_2$  stand for two real numbers on the real axis. The intervals defined for a specific value of the membership function, e.g.,  $\alpha = 0.5$ , represent the so-called  $\alpha$ -cuts. A fuzzy number can be equally expressed either by a nominal value and a membership function on each side of the nominal value or by a set of  $\alpha$ -cuts.

### Fuzzy arithmetic

A fuzzy arithmetic operation depends on the definition of a fuzzy number. In the cases when fuzzy numbers are defined by a set of  $\alpha$ -cuts, the problem of fuzzy arithmetic is reduced to the well-known arithmetic operations on intervals, which are applied to each  $\alpha$ -cut. In the case of real-life problems, such as dynamic analyses based on the finite element method, an extensive number of arithmetic operations are necessary and the formulation in the above terms is relatively expensive in terms of computational time. More can be found in, e.g. (Kaufman, 1985).

### **Fuzzy logic**

Fuzzy logic is reasoning with fuzzy sets. It extends the classical, bivalent, logic, which takes into account only the two states of truth and falsehood, to the multivalent logic, which can also consider all the intermediate states between the truth and falsehood. It can be viewed as a mathematical tool which can process vaguely defined values, such as “a little warmer”, rather than with an exact temperature difference, and yet deliver very exact rulings, such as “keep the heater off for 5 minutes”. In other words, fuzzy logic provides tools for mathematical description and subsequent solution of problems which are defined in natural language. Then, the quantities used in fuzzy reasoning are not exact real numbers, but fuzzy sets or fuzzy numbers in our case.

The basic algorithmic procedure for a fuzzy logic controller comprises those five major sequential steps: 1) definition of fuzzy sets in model, which in our case is selection of e.g. deflection and price of each constituent, etc. as relevant variables; 2) fuzzification of input and control variables, which in our case is construction of a fuzzy number over a range of possible, e.g., prices, see Figure 1; 3) knowledge base, which contains necessary dependencies and rulings, such as when a certain limit is exceeded, a corrective measure must be taken; 4) computational algorithm, which performs the rulings on the defined fuzzy sets, which yields the result in a form of fuzzy sets, or fuzzy numbers, and 5) defuzzification of fuzzy control outputs, which represents selection of a single real value which best represents the resulting fuzzy number as often a single value is desired when controlling some process. In our case, defuzzification of the resulting value was not performed as the information on possible distribution of prices would be lost and as the presented results can be readily used for other automated decision-making algorithms. More can be found in, e.g. (Ross, 1995).

### **METHOD OF ASSESSMENT**

Since all preliminary estimations have to deal with uncertainty contained in the input information, the uncertainty should propagate through the calculus till the end where the designer can assess its effect and make decisions accordingly. The method proposed is making use of the fuzzy sets theory and all quantitative information, which contains uncertainty, is expressed by fuzzy numbers. Figure 2 shows and an example of price of recycled concrete aggregate.

The flow of the assessment method is as follows; Firstly, the information on material properties, mix proportions of concrete and prices, which is in the form of fuzzy numbers, enters the design calculation, which is based on Eurocode 2. That means the commonly used design procedures with all the relevant mathematical formulas are processed using fuzzy arithmetic. In this way, each structural member is designed and checked for the ultimate and serviceability limit states while the system finds the optimal size of the cross-section with minimum price, when, for example, the ratio between width and height of a cross-section is set as a third to a half and the sizes must be a multiple of 5 to follow formwork requirements. The optimal cross-section is selected using the fuzzy-logic rulings which are based on the common designer's knowledge. The decision-making used for seeking the optimal sizes of cross-sections is based on the fuzzy-logic IF-THEN rules. That means there are fuzzy sets used in the algorithm which express the proximity of the solution toward the limit values, such as the maximum allowable deflection, and upon whose degrees of membership corresponding decisions are taken. At this stage the system offers all close solutions for each mix proportion, whose price range overlaps the minimum price range, so that the designer can further refine his preferences by imposing other conditions, such as minimum amount of cement, if the designer is environment-friendly oriented.

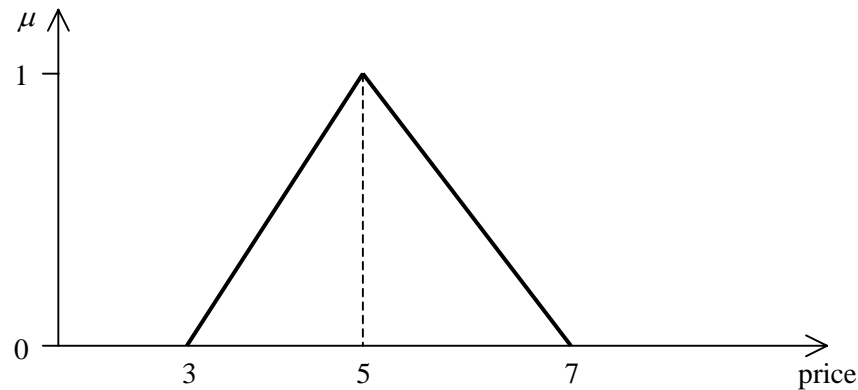


Figure 2: Example of estimation of price of recycled concrete aggregate (in Euro)

Secondly, the results for each structural member are then combined and the price of the entire structure is evaluated. Here, another fuzzy-logic ruling can be used for combining the structural members which ensures that sizes of connecting structural members correspond with each other, if required. This requirement, however, can increase price of the entire structure. In our example it was not employed. The system is designed so that it can work only with one type of concrete for the entire structure, which is quite realistic, instead of using different mix proportions for each structural member.

The result of the assessment is in the form for fuzzy distribution of prices, which is quite intuitive as the degree on the vertical axis expresses confidence in the respective interval of prices, see Figure 1, when the degree of 1 represents full confidences.

## EXAMPLES OF APPLICATION

The input parameters considered in the examples of application are given in Table 1, which comprises data on concrete made of recycled concrete aggregate published in (Topcu, 2004). The concrete mixes were prepared with a constant water-cement ratio and almost constant amount of cement. The percentage in the top row indicates the fraction of recycled concrete aggregate within the total amount of aggregate. The compressive strength and density was taken also from (Topcu, 2004). The modulus of elasticity was estimated with a formula from Eurocode 2 and the trend reflecting the percentage of recycled concrete aggregate from (Khatib, 2005). The creep coefficient was estimated from the basic knowledge. The lowest and highest prices of concrete made of natural aggregate (0%) were taken from the current pricelists of several Czech concrete produces. For other percentages, the price was calculated with respect to the price of natural stone and recycled concrete aggregate.

It is evident from Table 1 that if the designer opts for concrete made of only recycled concrete aggregate, the price range is well below the price range of concrete made of natural aggregate. The average price is about 66% of concrete made of natural aggregate, while the compressive strength is only 25% lower. That means the cost-performance ratio is higher for concrete made of recycled concrete aggregate already today. However, the designers are reluctant to use such a material as its ultimate performance is not fully predictable. In the least, the designer needs to take into account the increasing creep coefficient which signals problems with satisfying the criteria of the serviceability limit state.

Table 1: Input parameters for analyses

| Properties and prices of concrete | Amount of recycled aggregate |      |      |      |      |
|-----------------------------------|------------------------------|------|------|------|------|
|                                   | 0%                           | 30%  | 50%  | 70%  | 100% |
| Compressive strength (MPa)        | 18                           | 16.8 | 15.2 | 14.5 | 13.5 |
| Modulus of elasticity (GPa)       | 26.2                         | 24.9 | 23.6 | 22.3 | 21.0 |
| Density (kg/m <sup>3</sup> )      | 2358                         | 2326 | 2303 | 2274 | 2245 |
| Creep coefficient                 | 2.5                          | 3.0  | 3.3  | 3.6  | 4.0  |
| Lowest price (Euro)               | 61.5                         | 55.6 | 51.5 | 47.4 | 40.4 |
| Highest price (Euro)              | 70.7                         | 64.7 | 60.2 | 54.6 | 47.6 |

In order to quantify the effect of percentage of recycled aggregate, which substitutes the natural stone aggregate, the reinforced concrete design procedures available in Eurocode 2 are adopted. This however requires an assumption that the values of 28-day compressive strength given in Table 1 are the characteristic values, which can be accepted for our purposes.

### Effect of span and percentage of recycled aggregate on price of beam

In the first analysis, only average prices of materials are considered in order to obtain smooth curves for comprehensive comparison. The price of concrete is therefore taken as the average of the highest and the lowest price given in Table 1. The price of steel is taken as 0.80 Euro. Three basic spans of a reinforced concrete beam of 4, 5 and 6 metres, which are commonly encountered in reinforced concrete design, are selected for comparison with the corresponding uniform load of 10 kN/m, which is taken as constant. Only the self weight increases the total load.

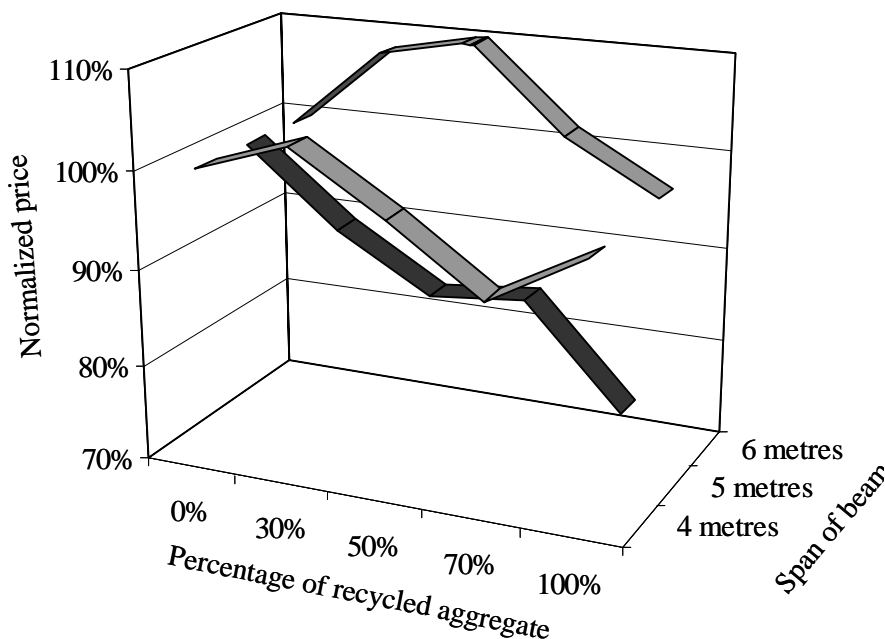


Figure 3: Effect of span and percentage of recycled aggregate on price of beam

According to Eurocode 2 design procedures, for the span of 4 metre the cross-sections varied from 350 x 200 mm to 400 x 250 mm (height x width), for 5-metre span it was 450 x 250 mm to 500 x 250 mm, and for 6-metre span 500 x 250 mm to 600 x 300 mm. It should be noted that the cross-section sizes are multiples of 50 mm. The serviceability limit state criterion on deflection was decisive in all cases. That means all cross-sections were unnecessarily large in order to satisfy the ultimate limit state condition of load-bearing capacity.

Figure 3 shows a comparison of the effect of percentage of recycled aggregate used on price of a reinforced concrete girder with the three different spans. For each span, the price of a girder made of concrete with natural stone aggregate, which still is being preferred for construction, is taken as the reference value, i.e. 100%. From the common designer's knowledge it is known that the optimum cost-performance ratio of a beam made of reinforced concrete is reached with the span of about 5 metres, which is caused by the increased price at shorter spans due to detailing restrictions and by the excessive effect of self weight at longer spans. That may explain why only the 5-metre-span curve is monotonically decreasing in Figure 3. The expected monotonic decrease in the 6-metre-span curve is disrupted by the sudden increases in sizes of the cross-section to satisfy the SLS criteria, which always in multiples of 50 mm due to formwork requirements, even though much less was necessary. In the case of the 6-metre-span curve, one also needs to take into account the extra 100 mm in the cross-section height, which eventually reduces the height of the ceiling, and to take into account the saved cost of building materials for reducing the height of all vertical structural members by the 100 mm.

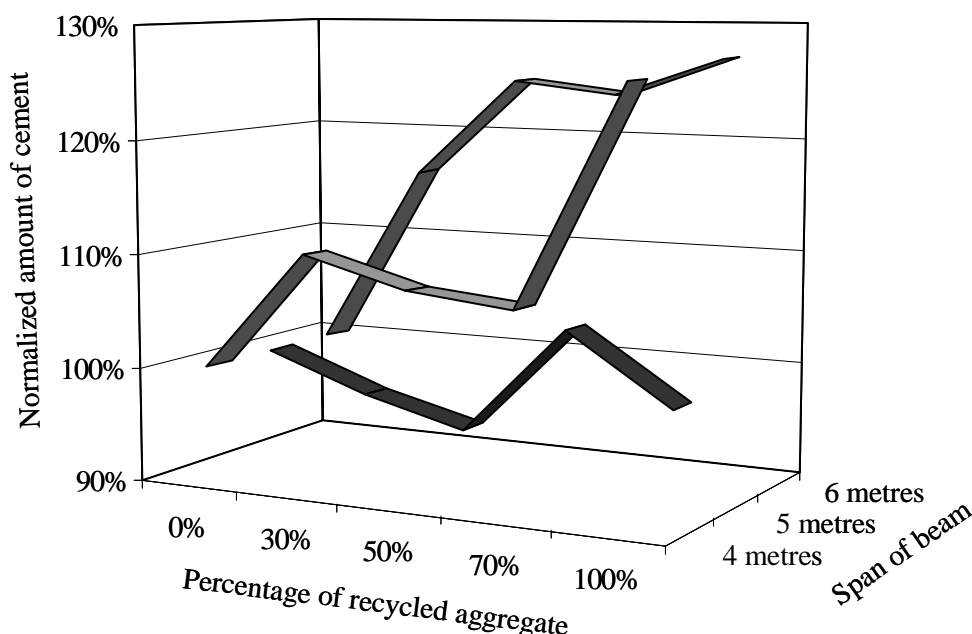


Figure 4: Effect of span and percentage of recycled aggregate on amount of cement used

For the true evaluation of sustainability of the beam, one should also account for the increased consumption of energies spent on production of building materials due to their higher amounts, which comes along with the larger cross-sectional areas. This effect is reflected in terms of the amount of cement used in Figure 4, which is related to the results shown in Figure 3. Also in Figure 4, the amount of cement is mostly lower with increasing amount of recycled aggregate only for the 5-metre span.

#### Automated selection of optimum mix proportions for lowest price of structural members

In this analysis, the fully automated algorithm for selection of the optimum mix proportions of concrete for the lowest price of particular structural members (girder, slab and column) and then the entire structure is used. Five different mix proportions are considered according to the percentage of recycled aggregate used in concrete, which are given in Table 1. In this case, prices of each mix proportion account for the uncertainty using the fuzzy numbers, where the minimum and maximum prices from Table 1 are set as the boundaries of the  $\alpha$ -cut for  $\alpha=0$  and their average is the price for  $\alpha=1$ . An example of such interpretation is shown in Figure 5. Standard steel B500C is considered in all analyses with its exact price of 0.80 Euro.

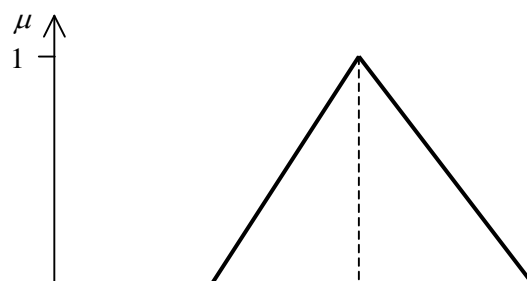


Figure 5: Example of estimation of price of concrete (in Euro)

For the analysis of a girder, live load of 12 kN/m, a span of 5.5 m and the maximum cross-section height of 550 mm are considered. The modular size of formwork is given in multiples of 50 mm.

Table 2: First six solutions for girder

| Ranking | Price of girder (Euro) |         |         | Percentage of recycled aggregate | Height x width (mm) | Amount of cement (kg) |
|---------|------------------------|---------|---------|----------------------------------|---------------------|-----------------------|
|         | Average                | Minimum | Maximum |                                  |                     |                       |
| 1       | 46.8                   | 44.0    | 49.5    | 30%                              | 550 x 200           | 214                   |
| 2       | 48.9                   | 46.2    | 51.6    | 70%                              | 550 x 250           | 259                   |
| 3       | 50.3                   | 47.0    | 53.5    | 100%                             | 550 x 300           | 293                   |
| 4       | 50.3                   | 47.6    | 53.1    | 0%                               | 550 x 200           | 221                   |
| 5       | 51.7                   | 48.6    | 54.9    | 30%                              | 500 x 250           | 243                   |
| 6       | 49.3                   | 52.6    | 55.9    | 50%                              | 550 x 250           | 262                   |

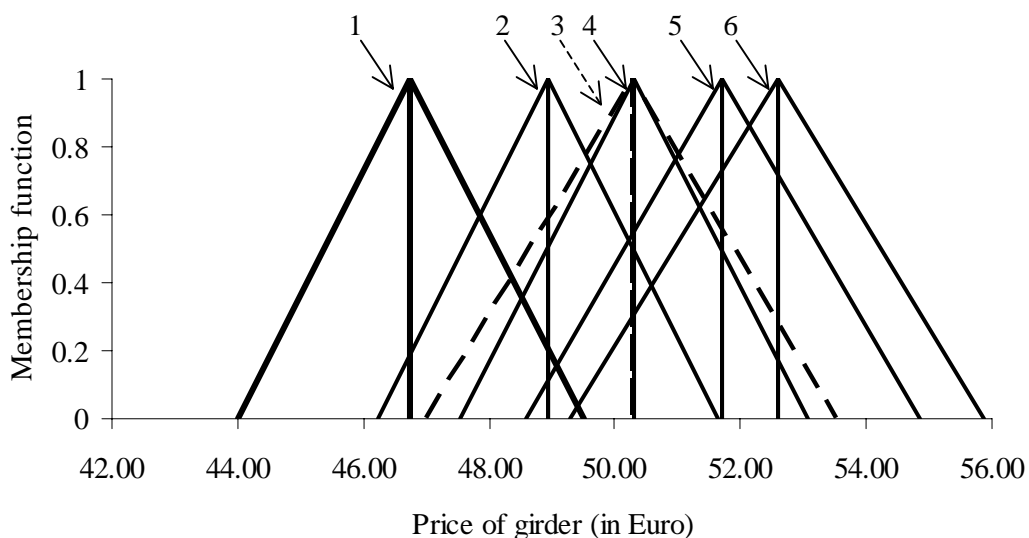


Figure 6: First six solutions for girder

Table 2 summarizes the first six best solutions, obtained by the algorithm explained above, whose price ranges overlap the minimum price range. The SLS criteria decided in all cases. The concrete with 30% of recycled aggregate yields the lowest price and the lowest amount of cement, however this solution takes extra 50 mm of the height of ceiling. The results are illustrated in Figure 6.

Table 3 contains the result of the analysis of a slab with a span of 2.5 m, when only the usual width of 1 m was considered in calculation. The SLS criteria decided in all cases. The load was 2 kN/m<sup>2</sup>. Here, the least expensive solution by far is that for concrete with 100% recycled aggregate. Only the first three solutions overlap. It should be noted that the average price of the slab made of natural aggregate concrete (0%) was 2.77 Euro higher than the best price which also means that the effect of

increased self weight due to the increased height of the slab can be neglected. The results are shown in Figure 7.

Table 3: First three solutions for slab

| Ranking | Price of slab (Euro) |         |         | Percentage of recycled aggregate | Height of slab (mm) | Amount of cement (kg) |
|---------|----------------------|---------|---------|----------------------------------|---------------------|-----------------------|
|         | Average              | Minimum | Maximum |                                  |                     |                       |
| 1       | 14.6                 | 13.6    | 15.6    | 100%                             | 110                 | 89                    |
| 2       | 16.4                 | 15.4    | 17.5    | 50%                              | 100                 | 87                    |
| 3       | 16.5                 | 15.5    | 17.5    | 70%                              | 110                 | 94                    |

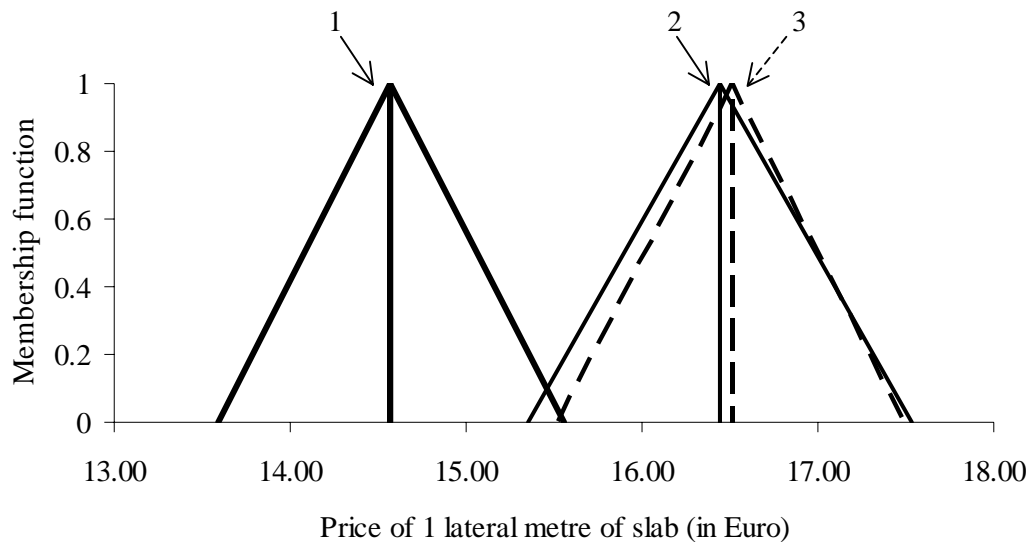


Figure 7: First three solutions for slab

In the analysis of a column, only the ultimate limit state conditions were used. The load considered was 2 MN and the height of the column was 4 metres. Another limitation imposed was a square cross-section. Here, the least expensive solution was that for the concrete with only natural stone aggregate, however the price for the cross-section made of concrete with 70% recycled aggregate was very similar. The first four best results are given in Table 4 and Figure 8. The solution for concrete with 30% recycled aggregate did not overlap the best solution.

Table 4: First four solutions for column

| Ranking | Price of column (Euro) |         |         | Percentage of recycled aggregate | Height x width (mm) | Amount of cement (kg) |
|---------|------------------------|---------|---------|----------------------------------|---------------------|-----------------------|
|         | Average                | Minimum | Maximum |                                  |                     |                       |
| 1       | 35.9                   | 33.7    | 38.2    | 0%                               | 350 x 350           | 179                   |
| 2       | 36.2                   | 33.9    | 38.5    | 70%                              | 400 x 400           | 220                   |
| 3       | 39.2                   | 36.3    | 42.1    | 100%                             | 450 x 450           | 262                   |
| 4       | 39.3                   | 36.5    | 42.1    | 50%                              | 400 x 400           | 221                   |

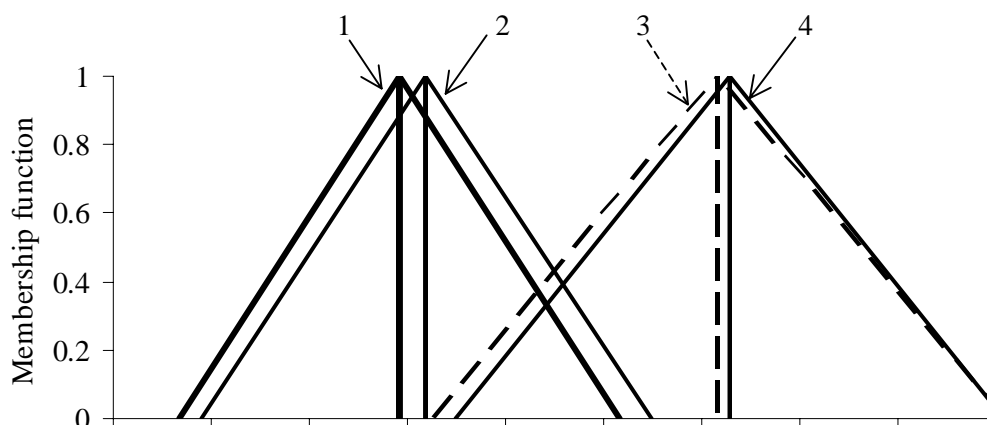


Figure 8: First four solutions for column

### Automated selection of optimum mix proportions for lowest price of entire structure

In this analysis, a floor structure consisting of typical structural members is considered. Since the same concrete is used in the entire structure, the overall minimum price of the structure is sought. That means the structural members with largest volume decide the mix proportions. Typically, it is the slab. The structure is shown in Figure 9 where all necessary sizes are given. The load considered, which is acting on the slab, is  $3 \text{ kN/m}^2$ . The load acting on columns was taken as  $2 \text{ MN}$  in order to simulate the effect of the upper floors.

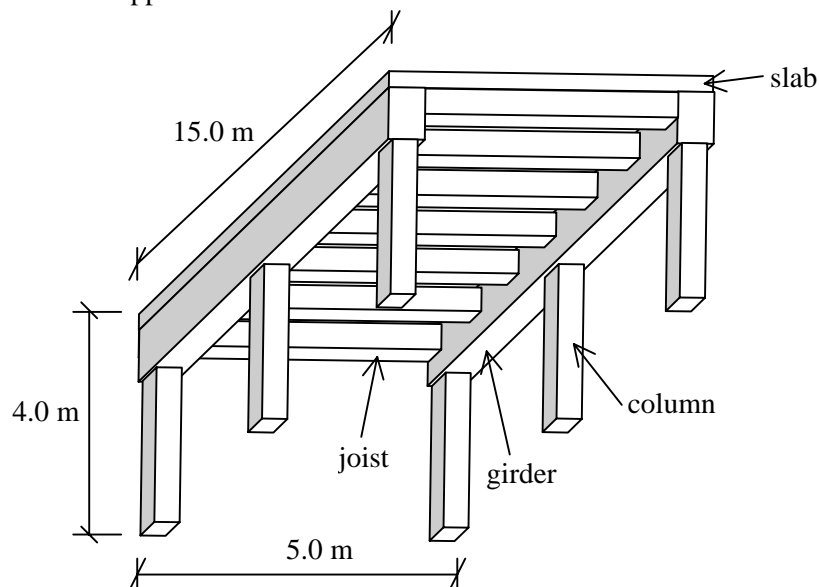


Figure 9: Analysed floor structure

The overall results are shown in Table 5 and Figure 10. The least expensive solution is that for concrete with 100% of recycled aggregate, however, it can be noted that the sizes of connecting structural members do not match, specifically the width of the girder and the cross-sectional sizes of the column. This condition was not required in this analysis. From Table 6 it is evident that the slab and the joists decided the price. The second best solution is that for concrete with only natural stone aggregate. If price was the only criterion, the 100% recycled aggregate variant would win. But, if the amount of cement used were also of an interest, the natural stone aggregate variant would be preferred. Also, the height of ceiling reduced by 10 centimetres should be considered. Also here, the serviceability-limit-state conditions, namely the deflection limitation, were decisive with the exception of the columns. Table 6 summarizes normalized prices and amounts of cement for each structural member, when the price was normalized with respect to the price of concrete with natural stone aggregate.

Table 5: First five solutions for structure

| Rank | Percentage of recycled aggregate | Price of structure (Euro) |      |      | Slab height (mm) | Joist cross-section (mm) | Girder cross-section (mm) | Column cross-section (mm) | Height of ceiling (m) | Amount of cement (kg) |
|------|----------------------------------|---------------------------|------|------|------------------|--------------------------|---------------------------|---------------------------|-----------------------|-----------------------|
|      |                                  | Aver.                     | Min. | Max. |                  |                          |                           |                           |                       |                       |
| 1    | 100%                             | 1269                      | 1184 | 1354 | 120              | 500 x 250                | 700 x 400                 | 450 x 450                 | 3.3                   | 8328                  |
| 2    | 0%                               | 1346                      | 1269 | 1424 | 100              | 450 x 200                | 600 x 300                 | 350 x 350                 | 3.4                   | 6789                  |
| 3    | 70%                              | 1347                      | 1268 | 1426 | 120              | 500 x 250                | 700 x 350                 | 400 x 400                 | 3.3                   | 8195                  |
| 4    | 50%                              | 1361                      | 1272 | 1449 | 110              | 450 x 250                | 650 x 350                 | 400 x 400                 | 3.35                  | 7691                  |
| 5    | 30%                              | 1399                      | 1310 | 1487 | 110              | 450 x 250                | 650 x 300                 | 400 x 400                 | 3.35                  | 7524                  |

Table 6: Prices and amounts of cement for each structural member of analysed structure

|                             |         | Percentage of recycled aggregate |      |      |      |      |
|-----------------------------|---------|----------------------------------|------|------|------|------|
|                             |         | 0%                               | 30%  | 50%  | 70%  | 100% |
| Normalized price            | Slab    | 100%                             | 100% | 94%  | 94%  | 83%  |
|                             | Joists  | 100%                             | 110% | 104% | 107% | 97%  |
|                             | Girders | 100%                             | 100% | 107% | 107% | 105% |
|                             | Columns | 100%                             | 115% | 108% | 98%  | 106% |
| Normalized amount of cement | Slab    | 100%                             | 106% | 104% | 112% | 106% |
|                             | Joists  | 100%                             | 121% | 118% | 130% | 123% |
|                             | Girders | 100%                             | 105% | 119% | 128% | 137% |
|                             | Columns | 100%                             | 124% | 122% | 119% | 142% |

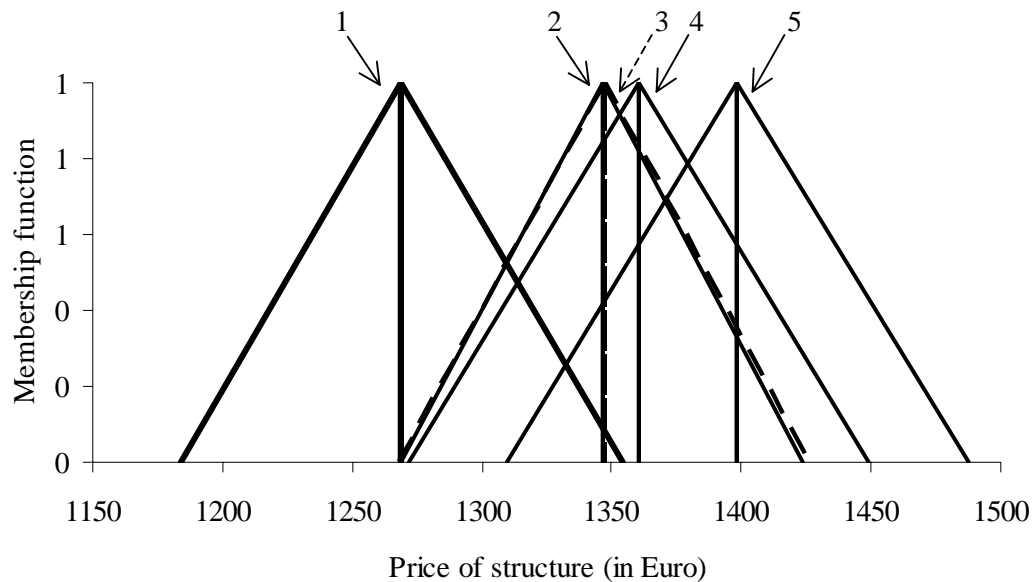


Figure 10: First five solutions for structure

It is also evident from Table 5 that with the exception of the second best solution for concrete with natural stone aggregate, when the better material properties of concrete, namely modulus of elasticity and creep coefficient, allowed reduction of cross-section sizes and thus reduced price, higher percentage of less expensive recycled aggregate yielded lower price of the structure.

## CONCLUSION

This paper presented a method for assessment of structures made of recycled aggregate. In order to cover the possible differences in prices of materials, which in the end affect the price of a structure, the easily comprehensible fuzzy numbers were used. The results of the presented analyses revealed that despite the reduced material properties of concrete with recycled aggregate, especially time-dependent properties, which resulted in more massive cross-sections, the lower price made concrete with recycled aggregate quite competitive already today. Therefore, possible increase in tax on waste and various subsidies would make recycled aggregate even more appealing. The only drawback in the process is the reluctance of designers to use recycled aggregate on a routine basis, which may be removed by simple methods providing quantitative information on performance of structures and their economical feasibility, such as the one presented in this paper.

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